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Electron-atomic-hydrogen elastic collisions in a laser field

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Abstract. The differential cross-section for electron-hydrogen atom collisions in the presence of a linearly polarized laser field is studied as a function of a scattering angle of low energy electron by employing second-Born approximation (SBA). Detailed analysis is performed in no forward scattering angle.

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1 Introduction

Laser-assisted electron-atom scattering offers the possibility of observing various multiphoton phenomena at relatively low laser intensities and as an important process in understanding stellar atmospheres, Laboratory discharges and plasmas. With the availability of lasers it has become possible to make detailed studies of these differential cross-sections, not only for single-photon exchanges, but also for multiphoton processes. The challenge for theory lies in accurately treating each of electron-target, electron-laser and laser-target interactions. Perturbative treatments may be used if one of these dominates the others. In an early work on free-free scattering, for example, Kroll and Watson [1] treated the laser-electron interaction with higher order terms in the Born series, while the dressing of the target by the field was neglected, to obtain a formula that is valid when the frequency ω of the laser field is much smaller than the kinetic energy of the incident electron. The experimental data concerning the large-angle scattering are in reasonable agreement with the Kroll-Watson-type approximations (KWA), which neglect the internal degrees of freedom of the atom. In this low-frequency approximation, the differential cross-section for free-free scattering is expressed as the product of the field-free differential cross-section evaluated at shifted initial and final electron momenta and a factor that depends on the field and the electron momentum transfer. the derivation of the KWA breaks down at critical geometries where the direction of the electric field is perpendicular to the momentum transfer, but the differential cross-sections for these geometries are expected to be very small [1].

Several experiments have been performed, in which the exchange of one or more photons between the electron-atom system and the laser field has been observed [2–7]. Moreover, the laser field introduces new parameters into the description of the collision such as its intensity, its frequency, and its polarization. At present, almost all the free-free experiments have been performed with a CO_2 laser as radiation field ($\hbar\omega = 0.117$ eV) and used helium and argon as atomic-target. For such cases a number of experiments have verified qualitatively the predictions of the KWA at large scattering angles [2]. In an early experiments on argon and helium targets, at critical geometries, where the laser polarization is almost perpendicular to the momentum transfer, Wallbank and Holmes [3–5] have however measured angular distributions larger by several orders of magnitude than those predicted by KWA. They suggested that the disagreement could be due to the polarization of the target by the field and/or its dressing effects (the effects of the internal degrees of freedom of the atom).

The aim of this work is to give new analysis about our previous works [8,9] in particular for no forward scattering angles where the most experiments were performed and the results are qualitatively agree with KWA.

The paper is structured as follows. In Section 2 we present the general formation of laser-assisted inelastic electron-atom collisions in the case of linear polarization of low-energy electrons. An account is then given of the techniques that we have used to evaluate the scattering amplitudes. Section 3 contains a detailed of our numerical results as well as their physical interpretation and interest. Unless otherwise stated atomic units (au) are used throughout.

2 Theory, results and discussion

The process in the course of which ℓ photons from the laser field are exchanged, while the inelastic (elastic scattering

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and excitation) collision takes place, can be described by the following equation:

$$e^-(\mathbf{k}_i) + \mathrm{H}(1s) + \ell\hbar\omega \longrightarrow \mathrm{H}(nl) + e^-(\mathbf{k}_f), \qquad (1)$$

which represents the collision of an incoming electron with momentum $\mathbf{k}_i$, which is incident on the hydrogen atomic-target (initially in its ground state) in the presence of an intense laser field. The field is treated classically as single mode and spatially homogeneous, which means that it varies little over the atomic range and that the dipole approximation is valid. Working in the Coulomb gauge, we have for the vector potential of a field propagating along the $\hat{\mathbf{z}}$-axis and represented in the collision plane $(\hat{\mathbf{x}} - \hat{\mathbf{y}})$

$$\mathbf{A}(t) = A_0 \left[\hat{\mathbf{x}} \cos(\omega t + \varphi) + \hat{\mathbf{y}} \sin(\omega t) \tan\left(\frac{\eta}{2}\right)\right], \qquad (2)$$

with the corresponding electric field

$$\mathcal{E}(t) = \mathcal{E}_0 \left[\hat{\mathbf{x}} \sin(\omega t + \varphi) - \hat{\mathbf{y}} \cos(\omega t) \tan\left(\frac{\eta}{2}\right)\right], \qquad (3)$$

where $\mathcal{E}_0 = \omega A_0/c$, $\mathcal{E}_0$ and ω are the peak electric field strength and the laser angular frequency, respectively. Here η measures the degree of ellipticity of the field and we have the particular cases of linear polarization ($\eta = 0$) and circular polarization ($\eta = \pi/2$) are easily recovered. Here φ denotes the initial phase of the laser field. We can recast the electric laser field in terms of its spherical components by

$$\mathcal{E}(t) = \mathcal{E}_0 \sum_{\nu=\pm1} i\nu\hat{\varepsilon}_\nu \exp(-i\nu(\omega t + \varphi)), \qquad (4)$$

where $\hat{\varepsilon}_\nu = (1/2)[\hat{\mathbf{x}} + i\nu\hat{\mathbf{y}} \tan(\eta/2)]$ is the unitary polarization vector.

The energy conservation relation corresponding to the laser-assisted inelastic collisions of equation (1) reads

$$E_{\mathbf{k}_i} + E_i + \ell\hbar\omega = E_f + E_{\mathbf{k}_f}, \qquad (5)$$

where E_i and E_f are, respectively, the ground and final state energies of the atomic hydrogen target, while $E_{\mathbf{k}_i} = \mathbf{k}_i^2/2$ and $E_{\mathbf{k}_f} = \mathbf{k}_f^2/2$ represent, respectively the kinetic energy of the incident and scattered electron.

The interaction between the projectile and the laser field is treated exactly and its solution is given by the non-relativistic Volkov wave function $\chi_p(\mathbf{r}_0, t)$ [10,11], where $\mathbf{k}$ is the projectile wave vector and r_0 represents the free electron coordinate.

For the laser-target interaction, since we are interested by fields which have electric strengths smaller than the atomic unit ($\mathcal{E}_0 \ll 5 \times 10^9\,\mathrm{Vcm}^{-1}$) and frequencies different from the atomic transition energies, then the perturbation theory is the most appropriate method to solve the interaction process. If one restricts oneself to the first order, the 'dressed' wave function $\Phi_n(\mathbf{r}, t)$ is well-known (see [8–11]). Here $\mathbf{r}$ is the coordinate of the hydrogen target electron and n is the principal quantum number.

Remembering that if we consider a collision kinematics, where the incident electron is fast and exchange effects are small, we shall, as a first approximation, carry out a first-Born treatment of the scattering process. The first-Born S-matrix element for the direct inelastic collision from the ground state of the target to a final state of energy E_f, in the presence of the laser field is given, by the expression

$$\mathbf{S}_{f,i}^{B_1} = -i \int_{-\infty}^{+\infty} dt \langle\, \chi_{\mathbf{k}_f}(\mathbf{r}_0, t)\Phi_f(\mathbf{r}_1, t) \mid V_d(r_0, r_1)| \times \chi_{\mathbf{k}_i}(\mathbf{r}_0, t)\Phi_i(\mathbf{r}_1, t)\rangle, \qquad (6)$$

where $V_d(r_0, r_1) = -1/r_0 + 1/r_{01}$ is the direct electron-atom interaction potential, with $r_{01} = |\mathbf{r}_0 - \mathbf{r}_1|$, $\chi_{\mathbf{k}_i}(\mathbf{r}_0, t)$ and $\chi_{\mathbf{k}_f}(\mathbf{r}_0, t)$ are respectively the Volkov wave functions of the incident and scattered electrons in the presence of the laser field. $\Phi_i(\mathbf{r}_1, t)$ and $\Phi_f(\mathbf{r}_1, t)$ are respectively the 'dressed' atomic wave functions describing the fundamental and final states. This type of contribution to different scattering processes has been previously computed in various instances [12–14]. By expanding the integrand in a Fourier series and integrating over t, we can recast equation (6) in the form

$$S_{f,i}^{B_1} = i(2\pi)^{-1} \sum_{\ell=-\infty}^{+\infty} \delta(E_{\mathbf{k}_f} + E_f - E_{\mathbf{k}_i} - E_i - \ell\omega)\, f_{f,0}^{B_1,\ell}(\mathbf{\Delta}), \qquad (7)$$

where ℓ is the number of photons emitted during the collision, so that positive values of ℓ corresponding to absorption and negative ones to emission and momentum transfer $\mathbf{\Delta} = \mathbf{k}_i - \mathbf{k}_f$. The first-Born scattering amplitude, $f_{f,i}^{B_1,\ell}(\mathbf{\Delta})$, is corresponding to the process $i \longrightarrow f$ accompanied by the transfer of ℓ photons, can be split in an electronic and an atomic amplitudes. They can be written as

$$f_{f,i}^{B_1,\ell}(\mathbf{\Delta}) = f_{elec}^{B_1,\ell}(\mathbf{\Delta}) + f_{atom}^{B_1,\ell}(\mathbf{\Delta}) \qquad (8)$$

with

$$f_{elec}^{B_1,\ell}(\mathbf{\Delta}) = -\frac{2}{\Delta^2} J_\ell(\lambda)\langle\, \psi_f(\mathbf{r}) \mid V_d(\mathbf{r}_0, \mathbf{r})|\psi_i(\mathbf{r})\,\rangle, \qquad (9)$$

$$f_{atom}^{B_1,\ell}(\mathbf{\Delta}) = f_1(\mathbf{\Delta}) + f_2(\mathbf{\Delta}), \qquad (10)$$

$$f_1(\mathbf{\Delta}) = -\frac{i}{\Delta^2} \sum_n \left(\frac{J_{\ell+l}(\lambda)}{\omega_{ni} + \omega} - \frac{J_{\ell-l}(\lambda)}{\omega_{ni} - \omega}\right) \times M_{ni}\langle\psi_f|\tilde{V}_d(\mathbf{\Delta}, \mathbf{r})|\psi_n\rangle \qquad (11)$$

and

$$f_2(\mathbf{\Delta}) = -\frac{i}{\Delta^2} \sum_n \left(\frac{J_{\ell-l}(\lambda)}{\omega_{fn} + \omega} - \frac{J_{\ell+l}(\lambda)}{\omega_{fn} - \omega}\right) \times M_{fn}\langle\psi_n|\tilde{V}_d(\mathbf{\Delta}, \mathbf{r})|\psi_i\rangle \qquad (12)$$

where

$$\tilde{V}_d(\mathbf{\Delta}, \mathbf{r}) = \exp(i\mathbf{\Delta} \cdot \mathbf{r}) - 1 \qquad (13)$$

J_ℓ is an ordinary Bessel function of order ℓ. The terms $f_{elec}^{B_1,\ell}(\mathbf{\Delta})$ and $f_{atom}^{B_1,\ell}(\mathbf{\Delta})$ are called, respectively 'electronic' (which correspond to the interaction of the laser field with the projectile only) and 'atomic' (which include the atomic dressing effects and thus describe the distortion of the target by the electromagnetic radiation). Here $M^{\pm}_{n'm} = \mathcal{E}_0\langle\psi_{n'}|\hat{\varepsilon}_{\pm}\cdot\mathbf{r}|\psi_m\rangle$ are the dipole coupling matrix elements, $\omega_{nn'} = E_n - E_{n'}$ are the atomic transition frequencies, $\lambda = \mathbf{\Delta}\cdot\alpha_0$ and ψ_n is a target state of energy E_n in the absence of an external field.

It should be noted that the sums over intermediate states appearing in the expressions (11) and (12) can be divided in two classes because of the selection rules arising from the matrix elements $M_{n,n'}$. Indeed, the first sum only involves intermediate states with angular momentum $\ell' = 0$; the second sum only involves intermediate states with the final angular momentum $\ell = \ell' \pm 1$, where ℓ' is the angular momentum of intermediate state.

The first-Born differential cross-sections, which accounts for the 'dressing' effects due to the dipole distortion of the target atom by the laser field and corresponds to the various multiphoton processes, are given by

$$\left(d\sigma^{B_1,\ell}_{f,i}/d\Omega\right) = \frac{k_f}{k_i}|f^{B_1,\ell}_{f,i}(\mathbf{\Delta})|^2, \tag{14}$$

where the amplitude $f^{B_1,\ell}_{f,i}$ is given by equation (8).

If only the 'electronic' term retained, which ignores the 'dressing' of the target (has the familiar form obtained by studying laser-assisted potential scattering in the first Born approximation (FBA)), the first-Born differential cross-section for elastic scattering and excitation process would be given by

$$\left(d\sigma^{B_1,\ell}_{f,i}/d\Omega\right)_{no\ dressing} = \frac{k_f}{k_i}|f^{B_1}_{f,i}(\mathbf{\Delta})|^2 J^2_\ell(\lambda), \tag{15}$$

where the quantity $|f^{B_1}_{f,i}(\mathbf{\Delta})|^2$ is just the field-free first-Born differential cross-section corresponding to the scattering process $(i,\mathbf{k}_i) \longrightarrow (f,\mathbf{k}_f)$.

It convenient to use the contribution of the higher order of the Born serie and exchange effects for the slow incident electron, in inelastic electron-atom process in the presence of a laser field. As an example, the second-order contribution to the S-matrix element for electron-atom collisions from the ground state to a final state of energy E_f, in the direction channel and in the presence of a laser field accompanied by the transfer of ℓ photons, can be given by

$$\begin{aligned} S^{B_2}_{f,i} &= -i\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \\ &\times \langle\chi_{\mathbf{k}_f}(\mathbf{r}_0,t)\Phi_f(\mathbf{r},t)|V_d(\mathbf{r}_0,\mathbf{r})G_0^{(+)}(\mathbf{r}_0,\mathbf{r},t;\mathbf{r}'_0,\mathbf{r}',t') \\ &\times V_d(\mathbf{r}'_0,\mathbf{r}')|\chi_{\mathbf{k}_i}(\mathbf{r}'_0,t')\Phi_i(\mathbf{r}',t')\rangle, \end{aligned} \tag{16}$$

where $G_0^{(+)}$ is the causal propagator. It should be noted that this term as it stands, is second-order in the electron-atom interaction potential V_d and contains atomic wave functions corrected to first-order in the laser field strength $\mathcal{E}_0$. If one retains a global first-order correction in $\mathcal{E}_0$ for the target "dressed" states, one finds that $S^{B_2}_{f,i}$ is the sum of two terms which are respectively of zeroth and first-order in $\mathcal{E}_0$.

The leading matrix element, $S^{B_2,0}_{f,i}$, describes the collision of a Volkov electron by the undressed atom, i.e., the second-order contribution to the S-matrix element for laser-assisted collisions of zeroth-order in $\mathcal{E}_0$ is approximated in terms of a simpler second-Born amplitude by

$$\begin{aligned} S^{B_2,0}_{f,i} &= -(2\pi)^{-1}i \\ &\times \sum_{\ell=-\infty}^{\ell=+\infty}\delta(E_{\mathbf{k}_f} - E_{\mathbf{k}_i} + E_f - E_i - \ell\omega) f^{B_2,\ell,0}_{f,i}(\mathbf{\Delta}), \end{aligned} \tag{17}$$

with

$$f^{B_2,\ell,0}_{f,i}(\mathbf{\Delta}) = J_\ell(\lambda) f^{B_2}_{f,i}(\mathbf{\Delta}), \tag{18}$$

where

$$\begin{aligned} f^{B_2}_{f,i}(\mathbf{\Delta}) &= -\frac{1}{\pi^2} \\ &\times \int_0^{+\infty} q^2 dq d\xi'_q \frac{\langle\psi_f|\widetilde{V}_d(\mathbf{\Delta}_f,\mathbf{r})G_c(\xi')\widetilde{V}_d(\mathbf{\Delta}_i,\mathbf{r})|\psi_i\rangle}{\Delta_i^2\Delta_f^2} \end{aligned} \tag{19}$$

is the filed-free second-Born inelastic amplitude evaluated at the shifted momenta $\mathbf{\Delta}_i$ and $\mathbf{\Delta}_f$. Here

$$G_c(\xi') = \sum_n \frac{|\psi_n\rangle\langle\psi_n|}{\xi' - E_n}$$

is the Coulomb Green's function with argument $\xi' = E_{\mathbf{k}_i} + E_i - E_{\mathbf{q}} + \ell\omega$.

In the same way, the contribution to the S-matrix element for laser-assisted collisions of first-order in $\mathcal{E}_0$, is given by shifting the pole of the integrand, respectively, below the real ω-axis by a small positive quantity $\varepsilon \longrightarrow 0^+$,

$$\begin{aligned} S^{B2,1}_{f,i} &= -(2\pi)^{-1}i \\ &\times \sum_{l=-\infty}^{\infty}\delta(E_{\mathbf{k}_f} - E_{\mathbf{k}_i} + E_f - E_i - \ell\omega)\, f^{B_2,\ell,1}_{f,i}(\mathbf{\Delta}), \end{aligned} \tag{20}$$

with

$$f^{B_2,\ell,1}_{f,i}(\mathbf{\Delta}) = iJ'_L(\lambda)\left[f_1(\mathbf{\Delta}) + f_2(\mathbf{\Delta}) + f_3(\mathbf{\Delta})\right], \tag{21}$$

where

$$f_1(\mathbf{\Delta}) = -\frac{1}{(2\pi)^2}\sum_{n,n'}\int d\mathbf{q}\frac{f^{B_1}_{f,n'}(\mathbf{\Delta}_f)\, f^{B_1}_{n',n}(\mathbf{\Delta}_i)M_{n,i}}{(E_q - E_{k_i} + \omega_{n',f} - i\varepsilon)\omega_{n,i}}, \tag{22}$$

$$f_2(\mathbf{\Delta}) = -\frac{1}{(2\pi)^2}\sum_{n,n'}\int d\mathbf{q}\frac{M_{f,n'}f^{B_1}_{n',n}(\mathbf{\Delta}_f)\, f^{B_1}_{n,i}(\mathbf{\Delta}_i)}{\omega_{n',f}(E_q - E_{k_i} + \omega_{n,i} - i\varepsilon)} \tag{23}$$

and

$$f_3(\boldsymbol{\Delta}) = -\frac{1}{(2\pi)^2} \times \sum_{n,n'} \int d\mathbf{q} \frac{f^{B_1}_{f,n'}(\boldsymbol{\Delta}_f)\, M_{n',n} f^{B_1}_{n,i}(\boldsymbol{\Delta}_i)}{(E_q - E_{k_i} + \omega_{n',f} - i\varepsilon)(E_q - E_{k_i} + \omega_{n,i} - i\varepsilon)}. \quad (24)$$

The study of second-order corrections to atomic s-p amplitudes show that these corrections tend to a constant value of order k_i^{-1} as Δ becomes small, i.e., at small scattering angle and thus are rather unimportant in this angular range. However, this is precisely the scattering angular region, which we interest for $\mathcal{E}_0 \ll 1$ au, because the first-order amplitude is adequate to provide a significant 'dressing' effects, which supply a contribution of order Δ^{-1} and thus rule the differential cross-section, while at larger scattering angles the target 'dressing' becomes less important, and under non-resonant conditions one also can model the atom by a structureless center of force. For this, in our previous works, we have neglected the second-order contribution to the S-matrix element for laser-assisted collisions calculated in first-order in $\mathcal{E}_0$ (for more detailed analysis, see Refs. [8,9]). When this approximation is adopted, we may concentrate our discussion on the computation of the dominant term, $S^{B_2,0}_{f,i}$, in laser-assisted collisions and its computation. Thus, the electron-atom interaction amplitudes with the transfer of ℓ photons may be written, in SBA, as

$$f^{\ell}_{f,i}(\boldsymbol{\Delta}) = f^{B_1,\ell}_{f,i}(\boldsymbol{\Delta}) + f^{B_2,\ell,0}_{f,i}(\boldsymbol{\Delta}), \quad (25)$$

where $f^{B_1,\ell}_{f,i}(\boldsymbol{\Delta})$ and $f^{B_2,\ell,0}_{f,i}(\boldsymbol{\Delta})$ are respectively, the first-order and second-order amplitudes, which are given by equations (8) and (18). In other case, i.e. when the scattering angle is not small, the S-matrix element contribution for laser-assisted collisions of first-order in $\mathcal{E}_0$, becomes significant and the equation (25) can be written in the form

$$f^{\ell}_{f,i}(\boldsymbol{\Delta}) = f^{B_1,\ell}_{f,i}(\boldsymbol{\Delta}) + f^{B_2,\ell,0}_{f,i}(\boldsymbol{\Delta}) + f^{B_2,\ell,1}_{f,i}(\boldsymbol{\Delta}), \quad (26)$$

where $f^{B_2,\ell,1}_{f,i}(\boldsymbol{\Delta})$ represents the first-order correction in $\mathcal{E}_0$ of the second-order scattering amplitude, which is given by expression (21).

The main problem in evaluating the scattering amplitudes corresponding to the second-order contributions to the S-matrix element for laser-assisted elastic scattering and excitation process, consists

(i) in performing the summation over the intermediate states. In order to calculate exactly the corresponding radial amplitudes without further approximation, we have used two different methods based in Sturmian approach similar to the ones described in our previous works [8,9]. The Sturmian approach allows us to take into account exactly the bound-continuum-state contributions, which are of crucial importance for electron impact excitation at intermediate energies. These methods of computation constitute an important advantage in the present context as compared to earlier ones relying on the closure approximation [15];

(ii) in the presence of the intermediate wave vector $\mathbf{q}$ in the argument of the Bessel function. Indeed, the integrals, in expressions (19), (22), (23) and (24), over the virtual projectile states $\chi_{\mathbf{q}}(\mathbf{r}_0, t)$ with wave vector $\mathbf{q}$ is prohibitively difficult, which is actually zero at some values of incident electron energy and accordingly for some values of scattered electron energy. Each of these possible intermediate transitions will be characterized by a resonance behavior, i.e the denominator of the matrix elements entering the exact formula equations (17) and (20) being close to zero. Instead, we shall overcome this difficulty by determining the exact upper boundary of the integrals (19), (22), (23) and (24) over the virtual projectile [8]. However, one can overcome this difficulty by choosing a particular geometry, namely the geometry in which the polarization vector is parallel to the direction of the incident electron. In the latter $S^{B2,0}_{f,i}(\boldsymbol{\Delta})$ and $S^{B2,1}_{f,i}(\boldsymbol{\Delta})$ can be easily evaluated by numerical integration [16].

In elementary atomic processes identical particles are expected on physical grounds to respond differently to a strong external driving field, the effects due to the particles identity (exchange effects) must become less significant. Basically, the different response to the external perturbation to some extent makes the particles distinguishable. It is well-known from field-free electron atom collision theory that exchange effects lose their importance when the velocity of the incoming electron is considerably larger than that of the atomic electrons. In this case, two identical particles are in quite different physical states. However, in electron-atom collisions, where free and bound electrons are present, a strong driving field should affect in a different way the dynamics of the various electrons, and thus a reduction of exchange scattering amplitude should take place. Below we give a derivation of this effect taking the electron-atom collision in the presence of a laser field as an example.

The contribution for laser-assisted inelastic collisions to the S-matrix of exchange scattering which leads to some conceptual difficulties but would not significantly alter the results of the present discussion. We have consider in the present paper only the landing term of $g^{\ell}_{f,i}$, the exchange amplitude for electron–atom collisions with the transfer of ℓ photons. It is known the exchange effects in collisions are important at low relative velocities, while the FBA is an essentially high-energy approximation. Thus, the FBA does not seem the best approximation to look into the effects we are interested in. In favor of the FBA there are, however, the relative simplicity of the analytical treatment. In the same way, we may calculate the exchange scattering matrix element in FBA and in presence of laser field,

$$\mathbf{S}^{B_1,exc}_{f,i} = -i \int_{-\infty}^{+\infty} dt \langle \chi_{\mathbf{k}_f}(\mathbf{r}_1, t) \Phi_f(\mathbf{r}_0, t)| - \frac{1}{r_0} + \frac{1}{r_{10}} |\chi_{\mathbf{k}_i}(\mathbf{r}_0, t) \Phi_i(\mathbf{r}_1, t)\rangle, \quad (27)$$

Evaluating the time integration yields,

$$S_{f,i}^{B_1} = i(2\pi)^{-1} \sum_{\ell=-\infty}^{+\infty} \delta(E_{\mathbf{k}_f} + E_f - E_{\mathbf{k}_i} - E_i - \ell\omega) g_{f,0}^{B_1,\ell}(\mathbf{\Delta}), \tag{28}$$

where $g_{f,0}^{B_1,\ell}(\mathbf{\Delta})$ is the first-Born exchange scattering amplitude corresponding to the process $i \longrightarrow f$ accompanied by the transfer of ℓ photons. For this, in the case of the elastic scattering the exchange scattering amplitude is done in the closure approximation by using reference [17] and within. On other hand, in the case of excitation when the analytical calculations becomes non easy, we shall approximate the first-Born exchange amplitude for laser-assisted inelastic scattering by its dominant part, coming from the interaction term $(|\mathbf{r}_0 - \mathbf{r}_1|)^{-1}$, the scattering potential and its given by [8,9]

$$\begin{aligned} g_{f,i}^{B_1,\ell} \simeq & -\frac{1}{2\pi} \sum_{\ell'=-\infty}^{\ell'=+\infty} (i)^{\ell'} J_{\ell-\ell'}(\lambda) \int d\mathbf{r}_0 d\mathbf{r}_1 \exp(-i\mathbf{k}_f \cdot \mathbf{r}_1) \\ & \times \psi_i(r_0) \big\{ J_{\ell'} \big[\mathbf{a}_0 \cdot (\mathbf{r}_0 - \mathbf{r}_1) \big] |\mathbf{r}_0 - \mathbf{r}_1| \big\} \\ & \times \psi_i(r) \exp(i\mathbf{k}_i \cdot \mathbf{r}_0), \end{aligned} \tag{29}$$

with $\mathbf{a}_0 = \omega\alpha_0$ in au.

Thus, when exchange effects are to be considered in laser-assisted electron-atom collisions, the first-order amplitude $f_{f,i}^{B_1,\ell}(\mathbf{\Delta})$ is replaced by

$$f_{f,i}^{B_1,\ell}(\mathbf{\Delta}) + g_{f,i}^{B_1,\ell}(\mathbf{\Delta}), \tag{30}$$

where $g_{f,i}^{B_1,\ell}(\mathbf{\Delta})$ may be written in the form

$$g_{f,i}^{B_1,\ell}(\mathbf{\Delta}) \simeq J_\ell(\lambda) g_{f,i}^{Och}(\mathbf{k}_i, \mathbf{\Delta}) \ , \tag{31}$$

with

$$\begin{aligned} & g_{f,i}^{Och}(\mathbf{k}_i, \mathbf{\Delta}) = \\ & \quad -\frac{2}{\Delta^2} \int d\mathbf{r} \exp(i\mathbf{\Delta}.\mathbf{r}) |\psi_i(\mathbf{r})|^2 = \frac{\Delta^2}{k_i^2} f_{f,i}^{B_1}(\mathbf{\Delta}) \end{aligned} \tag{32}$$

is the exchange amplitude in Ochkur approximation. Here $f_{f,i}^{B_1}(\mathbf{\Delta})$ is the field-free first-Born amplitude for $i \longrightarrow f$ transition process.

The second-Born differential cross-section corresponding to the various multiphoton processes, with the transfer of ℓ photons, are given by

$$\left(\frac{d\sigma_{f,i}^\ell}{d\Omega} \right) = \frac{k_f}{k_i} \left[\frac{1}{4} |f_{f,i}^\ell + g_{f,i}^{B_1,\ell}|^2 + \frac{3}{4} |f_{f,i}^\ell - g_{f,i}^{B_1,\ell}|^2 \right] \tag{33}$$

does not depend on the initial phase φ of the laser field, due to the inability of the collision time to be defined, as a result of the approximation of the projectile wavepacket by a monoenergetic beam of infinite duration [18].

When comparing theoretical results to the experimental cross-sections, one first has to obtain cross-sections over a fine mesh of intensities and then convolute them with a realistic spatiotemporal distribution of intensities of the laser beam. The ponderomotive acceleration of the projectile when penetrating and leaving the interaction region should also be taken into account, unless the pulse is extremely short.

As an application of our results (33) we consider the case of the electron-atomic hydrogen inelastic (elastic and excitation process) collision. Our detailed calculations are evaluated for a geometry in which the polarization vector of the field $\mathcal{E}$ is parallel to the direction of the momentum transfer $\mathbf{\Delta}$, varying thus with the scattering angle θ, and with the number of photons transferred in the collision. The reason we adopt this geometry is that the angular part of the scattering amplitude may be simplified and, what is more important, because for small momentum transfers, when approximately $\mathbf{k}_i \perp \mathbf{\Delta}$, the coupling of the colliding system with the field has its minimal value for $\mathcal{E}_0 \parallel \mathbf{k}_i$, and its maximal value for $\mathcal{E}_0 \parallel \mathbf{\Delta}$. Although in a realistic experiment the choice of the geometry $\mathcal{E}_0 \parallel \mathbf{\Delta}$ causes inconveniences because of the necessity of rotating the laser beam for each ℓ and θ, the data concerning the experimental measurements of the elastic-electron helium-atom collision for this geometry have been recently reported [3]. Moreover, in the case of small-frequency, i.e. small-momentum transfer collisions, the results referring to the geometry $\mathcal{E}_0 \parallel \mathbf{\Delta}$ should be very close to those obtained for $\mathcal{E}_0 \perp \mathbf{k}_i$. Note, however, that through simplified, the model contains all ingredients needed for the discussion, our results are analyzed by estimating the first and second-Born differential cross-sections, where the electric field strength is kept fixed at 10^8 Vcm^{-1}(0.02 au).

In Figure 1 we present the differential cross-sections corresponding to the elastic electron-atomic hydrogen scattering with no net transfer of photons ($\ell = 0$), as a function of the scattering angle θ and for incident energies $E_{\mathbf{k}_i} = 20$ eV. According to the domain of validity of the treatment used for taking into account the laser-atom interaction, the Nd-NAG laser frequency will be taken to be $\hbar\omega = 1.17$ eV (0.043 au). These parameters are such that the target dressing gives the main contribution. For small scattering angle, i.e. small momentum transfer Δ, the differential cross-section corresponding to the electronic term much smaller than differential cross-section without laser field for $|\ell| \geq 1$ and the soft photon approximation is not valid. More precisely, we consider cases in which the energies of the 'projectile' electrons are the same as the ones for the field-free case [then $\ell = 0$ in the energy conservation relation, Eq. (5)]. It then makes sense to compare the differential cross-section's, computed within the SBA and FBA, for the laser-assisted and field-free collisions. The modifications of the cross-sections then directly reflect the role of the dressing of the projectile-target system by the external laser field. We are working in linear polarization where the laser-assisted differential cross-section only depends on the orientation of the polarization unit vector $\hat{\epsilon}$ [14]. The results obtained by using the SBA with exchange effects included (Eq. (33)) are compared to the total and electronic cross-section in FBA, and to the corresponding results without laser fields.

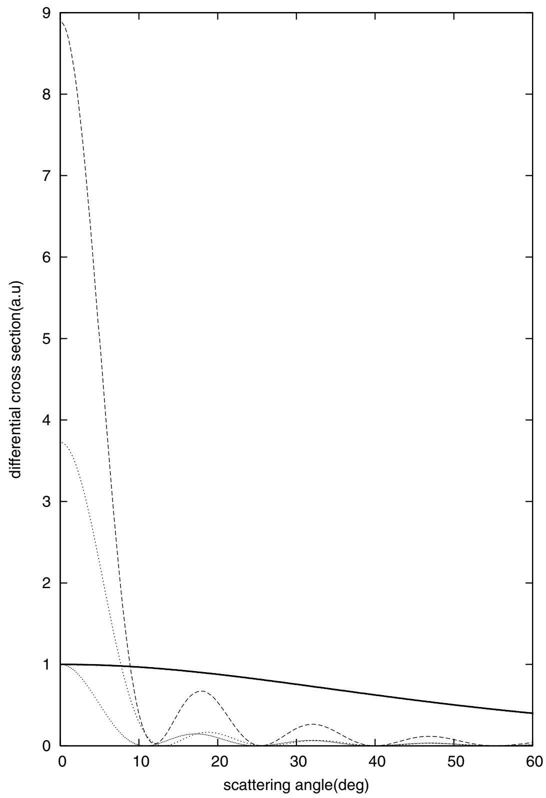

Fig. 1. Differential cross-section for elastic scattering with the transfer of no photon as a function of scattering angle (θ). The incident electron energy is 20 eV, the laser frequency is 1.17 eV and the electric field strength is 10^8 Vcm^{-1}. (- - - -) Second-Born approximation; (. . . . . . .) first-Born approximation, (.......) results obtained by neglecting the dressing of the target; (——) laser-off results.

As noted before the inclusion of higher order terms of the direct scattering matrix and of exchange only increases the results at small angles, where the differential cross-sections being the overall change in their relative magnitudes, in particular for the SBA. This is one interesting typical signatures of the dressing of the electron-target system in the differential cross-section and what clearly shows the effects of internal structure of the atomic target when the energies of the primary electron is weak. Then suggest taking fully into account the target distortion induced by a laser field. Such a distorted atom also acts on the projectile by a long-range dipole potential ($\sim 1/r^2$), which requires a non-perturbative treatment of laser-atom interaction. The long-range dipole potential affects mainly the distant collisions, which contribute to near forward scattering. For collisions at larger scattering angles (until some scattering angles) the target dressing remain important because of the presence of second-term of Born series in our results.

This point is, also illustrated in the set of Figures 2a and 2b, in which we show the influence of changing the photon number, everything else being kept fixed, on the scattering angular distribution. As shown in Figures 2a and 2b, the laser assisted differential cross-section in SBA

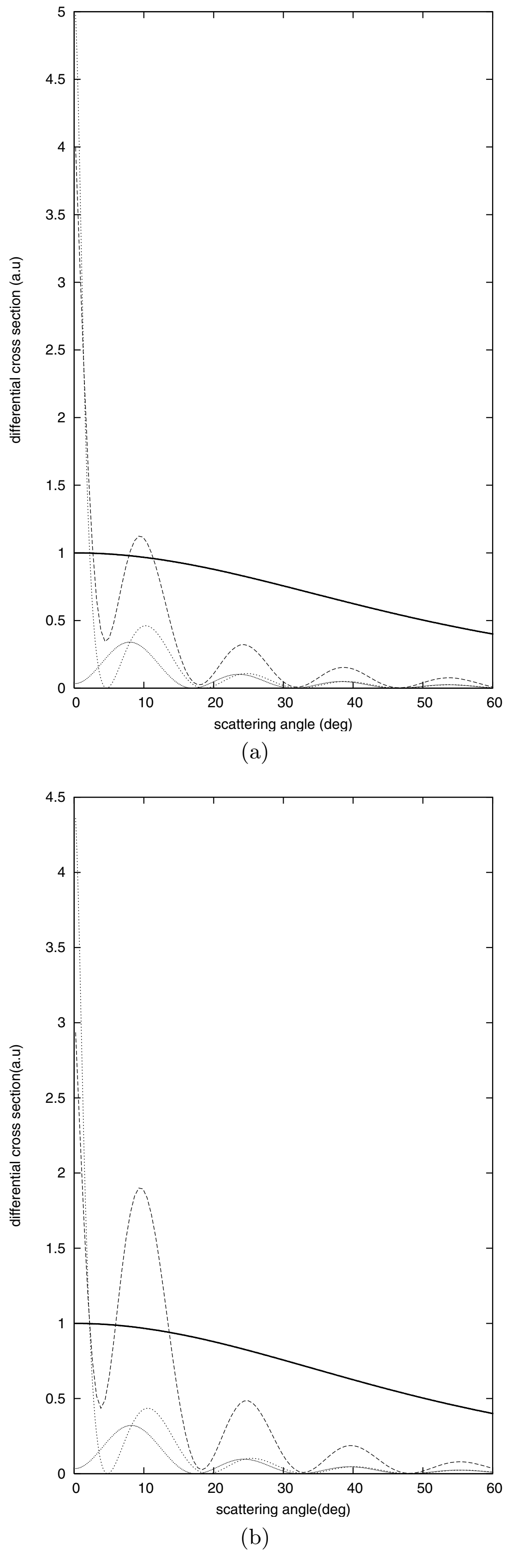

Fig. 2. (a) As Figure 1, but with the transfer of one photon $\ell = 1$. (b) As (a), but for emission of one photon $\ell = -1$.

deviates very little from the FBA one at small scattering angle below 3°. At higher scattering angle however, significant changes take place, as shown in Figures 2a and 2b, which are obtained for an absorption and emission of one photon, respectively. The angular distribution is then modified as the absorption and emission are now split with different magnitudes. Several interesting points, characteristic of this class of laser-assisted collisions, can be made at this stage.

(i) The presence of the laser breaks the regular behavior of the angular distribution. This remains true even for the SBA and with/without transfer of laser photons. Accordingly, besides the dynamical polarization of the target states [change of the interior structure of the atom]; the projectile states are also modified, the overall of the regular behavior with respect to the scattering angle being lost.

(ii) The magnitude of the cross-section for no net exchange of photons is significantly smaller than in the field free case. This results from the fact that the laser itself does not contribute to the elastic scattering. In fact, the laser redistributes the interior target structure and the diffused electrons in new channels associated to indices $\ell \neq 0$ in the energy conservation relation (5), which are accessible in the 'dressed' continuum of the atomic target (appearance of an intermediate structure of the continuum more ordered).

The role of the laser field strength intensity in the SBA for the net absorption and emission of one photons is represents in Figures 2a and 2b. These figures show that the laser-assisted differential cross-section differs markedly from the field free one. This behavior can be traced back to the fact that the argument of the Bessel functions $J_\ell(\lambda)$, entering the expressions of the amplitudes (9), (11), (12), (18), (21) and (24), grows with $\mathcal{E}_0$ and varies with the scattering angle. Accordingly, at higher field strengths, every other laser parameter being fixed, one explodes more zeros of the Bessel functions when varying the scattering angle. This clearly gives rise to the observed increase in the number of minima.

Another interesting point is the fact that the overall magnitude of the cross-section corresponding to the difference between SBA and FBA at no forward scattering angles increases and it becomes very significant for $4° \leq \theta \leq 18°$. Although this trend is expected for laser-assisted processes from the leading matrix element $S_{f,i}^{B_2,0}$ it is by no means easily deduced from the structure of the transition amplitude equations (23) and (24) (in fact, the opposite trend is observed for $\ell = 0$, where the amplitudes are dominated by the Bessel function $J_0(\lambda)$). Again, the magnitude of the argument of the Bessel functions $J_\ell(\lambda)$ is directly proportional to $\mathcal{E}_0$ and, contrarily to $J_0(\lambda)$, which decreases from unity when $|\lambda|$ increases, the other functions $J_\ell(\lambda)$ start to increase from zero and grow as $|\lambda|^\ell$. Although this argument cannot account for the details of the variations of the differential cross-section in FBA and in SBA with the laser field it provides a far estimate of the overall changes observed with respect to field-free.

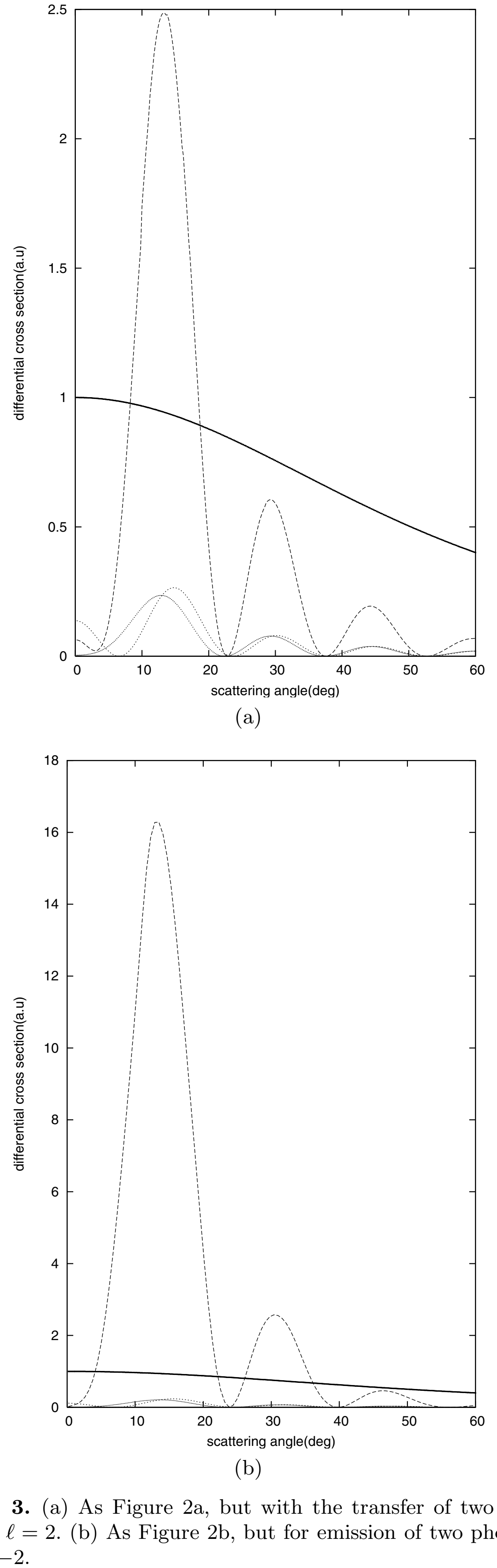

Fig. 3. (a) As Figure 2a, but with the transfer of two photons $\ell = 2$. (b) As Figure 2b, but for emission of two photons $\ell = -2$.

The set of Figure 3 also shows the inadequacy of the simplified approaches in which one neglects the dressing of the atomic targets by the field. Comparing again our results of SBA with those obtained with the FBA, on observes that, although the shape of the angular distribution is fairly well reproduced, the overall magnitude of the differential cross-section is significantly underestimated. In addition, one observes that, contrarily to the FBA the discrepancies are notable in large scattering angle when the second-term of the Born series is held account in the differential cross-section computations. This clearly shows that the collision dynamics is strongly affected by the dynamical polarization of the atomic target at large scattering angles. Thus, these tendencies are even amplified when turning to higher net number of exchanged photons. The most remarkable fact, however, are the important changes of magnitudes of the DCS with the second-order correction of atomic *s-p* amplitudes.

A second-Born approximation, for a given electric strength and laser photon energy, $\ell = 0$ cross-sections are two orders of magnitude larger for scattering in the forward direction than for $\ell = \pm 1$, due to the strong *s-p* coupling at small scattering and for low incident electron energies. Indeed, in the forward direction the dominant contribution to the $\ell = 1(-1)$ cross-section gives the terms corresponding to the absorption (emission) of one photon and a successive interaction with the projectile, and vice versa through the intermediate states. While the last term is slightly smaller than the former one for the $\ell = 1$ cross-section, it is larger in the case of the $\ell = -1$ cross-section due to the strong resonant coupling of ns and $n'p$ states. This coupling becomes too strong when the incident electron energies are weak, from where utility of the higher order of the Born series. The SBA term is added to the two terms of the FBA to interfered constructively in both cases. The term coming from the SBA is significant for $\ell = 0$ through the Bessel function what is not the case for $|\ell| = 1$. The situation is different at $\theta \neq 0$.

At present, almost all the free-free experiments have used CO_2 laser field and argon/helium as atomic target. For such cases the Kroll-Watson sum rule is valid. In general, the experimental measurements of the differential cross-section are taken in the range of the scattering angle non close to the forward direction [6]. Moreover an experimental program to study the free-free collusion from atomic hydrogen will require the collaboration of several experimental groups if they are to be successful [2]. For that, we had some results where the dressing effects are significant for the scattering angle close to zero and we believe that our results should serve as an incentive to perform such laser-assisted collisions experiments. Indeed, beyond the mere verification of the theory, such experiments would provide interesting information about the internal structure of dressed atoms, much in the spirit of ordinary atomic spectroscopy.

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